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# SOME TABLES OF THE NEGATIVE BINOMIAL DISTRIBUTION AND THEIR USE

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## PREFACE

In this Memorandum the author presents probability tables of the negative binomial distribution for some useful sets of parameter values. This distribution may be used as a frame of reference for the study of the demand for replacement parts.

## SUMMARY

This Memorandum presents tables giving the values of the individual terms of the negative binomial distribution for 130 pairs of parameter values in Part 1. Part 2, giving the cumulated terms of the same distributions, is in a form directly usable for solving problems which require the determination of probabilities of the occurrence of not more than x events.

The negative binomial distribution is described and illustrative examples are given which use this distribution as a tool in the study of demand for replacement parts.

The references indicate that the negative binomial has been used in a wide variety of applications to biological research.

# SOME TABLES OF THE NEGATIVE BINOMIAL DISTRIBUTION AND THEIR USE

## 1. INTRODUCTION

The research worker in the field of logistics is frequently required to predict the demand for spare parts in order that effective decisions can be made in the areas of procurement, distribution, and stockage policies. Many such decisions are made on the basis of a specialized knowledge about an individual line item.

For routine decisionmaking on a large scale, it is useful to have a small number of known probability distributions which can be used as a framework to provide the basic assumptions from which predictions of future demands can be made. If demand data are available, an analyst may study the sample data and make estimates of the parameters of the underlying demand distribution. The information in the sample will often be scanty, e.g., six months experience at one base or three months of system-wide experience. A frame of reference is needed to study the limited data.

Some examples of studies in which it was necessary to make an assumption about the demand for spare parts in the population may be found in the study of procurement deferral by Petersen [1], the design of flyaway kits by Karr, Geisler, and Brown [2], and by Fort [3], the prediction of

demands by Goldman [4], and Geisler and Brown [5], the study of stockage policies by Ferguson [6], Ferguson and Fisher [7], and Petersen and Geisler [8].

There are many distribution functions which could be discussed. Only two are presented here, the Poisson and the negative binomial. These two were chosen because they are probability distributions of a discrete variable, and demand for spare parts is by nature integral, and also because we believe, hopefully, that they may describe the universe of demands from which the sample emanates. These distributions are attractive also because of their easy computation.

## 2. THE POISSON DISTRIBUTION

When we deal with phenomena involving events that occur randomly in time or particles that are randomly distributed in space, the Poisson process is the model used. An experiment is performed and "events" are tallied. These events can be described by a function $x = x(t)$, which gives the number, $x$, of events observed during the first $t$ units of observation for all values of $t$ from 0 through $T$ (where $T$ is the total number of observations). The results of such an experiment may be described by the Poisson distribution if the events occur randomly in the sense of the following definition. If any number of events $x$ are observed in any amount of time $t$, and if the points of the occurrence of the $x$ events are independently and uniformly

distributed between 0 and t, then the process may be described as random. If the probability of the event is small but a large number of independent cases are taken, the number of occurrences is likely to be distributed in the Poisson series. This distribution may be thought of as an approximation to the binomial distribution when the probability of occurrence of the event is small, that is, if Np is large relative to p and N is large relative to Np (where N is the number of trials and p is the probability of the occurrence of the event in a single trial).

Let us try to visualize what the situation might be in regard to demand for spare parts upon the Air Force Supply System. Suppose a mechanic inspects an actuator on each of the 18 planes of a squadron, once each month for two years, and requests a replacement part whenever he judges that the actuator has failed. Assume that the failure rate for this part has been found to be 25 in 1000 (i.e., 1 in 40). We may then think of this situation as being represented by a binomial distribution $(p+q)^N$, where $N = 432$ (18 inspections per month for 24 months), $p = .025$, $q = .975$ (i.e., $q = 1-p$, the probability of nonoccurrence). But the data available to us is monthly data by squadron, and we are interested in determining a squadron demand rate per month. The monthly demand data for the actuator looks like this: 0-2-0-0-1-0-0-0-1-0-0-1-0-0-2-0-0-1-1-0-0-1-0-1. These numbers are ordered in time, the first observation being January, 1956 and the last one December, 1957.

This is a "natural" for the use of the Poisson approximation to the binomial, since Np is large relative to p (432 to 1) and N is large relative to Np (40 to 1). If we assume that the Poisson law describes the data and use as the parameter of the Poisson the mean demand per squadron month, i.e., $18 \times .025 = .45$, we will find a satisfactory fit with 15 months of no demand, 7 months of demand for one part, and 2 months of demand for 2 parts.

The Poisson (like the binomial) is a distribution of a discrete variable arising from enumeration data using integral values only. Its basic characteristic is the uniform probability of the occurrence of the event. In contrast to both the binomial and the normal distribution, it is defined by a single parameter, the mean. The variance is equal to the mean. The Poisson density is represented by the probability function $P(x) = e^{-m} m^x/x!$, where $x = 0,1,2,3\cdots$, and $m$ = mean.

Tables of the Poisson distribution have been widely published; for example see [9], [10], [11].

## 3. THE NEGATIVE BINOMIAL DISTRIBUTION

But the aircraft demand data described in Sec. 2 does not present this picture for many of the parts, and the Poisson distribution often has not been a good fit for the series of observations. In many cases, the lack of fit was manifested in more months of zero demand than that described by the Poisson distribution. It was also true

that we were getting more variation than was permitted under the Poisson assumption. For these reasons and others that will be discussed later, we used a distribution known as the negative binomial to fit the observed data. It is a two-parameter distribution of a discrete variable.

The following example illustrates this distribution. The monthly demand data from a squadron of aircraft for a door assembly shows the following series of demands over a 36-month period: 0-1-0-0-0-0-0-0-0-4-0-0-, 0-0-0-1-0-0-0-0-0-0-3-0-, 0-0-0-0-1-0-0-0-0-2-0-0. The sum of the demands is 12. The mean demand per month is one-third. The assumption of a Poisson distribution, using 1/3 as the parameter value, does not give a good fit to the data. The negative binomial distribution, using the calculation of moments of the observed distribution to estimate the parameters, gives a good fit to the data. The ratio of variance to mean is about 2.4. If we use an arbitrary ratio of variance to mean of 3, the fit is also good. The interpretation of "good fit" means that the amount of discrepance between the observed values and the theoretical values based on the assumed distribution is not large enough to indicate the presence of anything more than the caprices of random sampling. In other words, the hypothesis is upheld that this data could have been obtained from a population in which monthly demand was described by a negative binomial distribution.

The negative binomial distribution is completely defined by two parameters, the arithmetic mean m and a positive exponent k. The distribution is written $(q - p)^{-k}$ where $p = m/k$ and $p + 1 = q$. The general term in the expansion of this binomial gives the probability P that an observation x will have values 0, 1, 2, $\cdots$. The general term may be written

$$P(x) = \frac{(k+x-1)!}{(k-1)!x!} \frac{p^x}{q^{k+x}}, \quad x = 0,1,2,\ldots, P, k > 0.$$

The curve defined by the value of P(x) is unimodal, so that in fitting the negative binomial to an observed distribution, any apparent bimodal or multimodal tendency is attributed to random sampling. The negative binomial is an extension of the Poisson series in which the population mean m is not constant but varies continuously in a distribution which is proportional to that of Chi-square (The distribution referred to is called Pearson Type III or Gamma distribution). Thus the negative binomial may be used to represent a composite of several Poisson distributions in which the number of observations per unit time in repeated counts cannot be assumed to have the same expected valu  .mean) in each unit of measurement. Student [12] wrote in 1919 as follows: "If the presence of one individual in a division increases the chance of other individuals falling into that division, a negative binomial will fit best, but if it decreases the chance, a positive

binomial." Bliss reported [13] on fitting the negative binomial to biological data: "The negative binomial is the easiest to compute and the most widely applicable of the distributions for over-dispersion."

The negative binomial has been used in a variety of applications to biological data. It was used by Bliss [13] to fit a distribution function to counts of red mites on apple leaves. It was used by Morgan et al [14] to describe the distribution of bacterial clumps over a milk film. Student [15] used it for a description of the counting of yeast cells with a haemocytometer. Stirritt [16] et al. found that it was adequate to describe the distribution of corn borers in a field experiment. Greenwood and Yule [17] used a negative binomial to describe the distribution of accidents experienced by machinists. (The theory was that some machinists were more accident-prone than others. There was also the possibility that shop conditions differed from week to week in such a way as to cause accident risks to vary during successive weeks of the period covered by the data. In either case, the resulting distribution might be expected to be of the form of a negative binomial.) In plant ecology, quadrant counts which deviated from the Poisson distribution were attributed to the occurrence of plants in "clumps." Blackman [18] found that the distribution of plants per quadrant agreed very well with a negative binomial distribution, with estimates of parameters derived from the sample data. Jones, Mollison, and

Quenouille [19] used the negative binomial in fitting counts of soil bacteria. Sichel [20] made use of a negative binomial in studying psychological data on the occurrence of minor accidents in an industrial plant.

Additional references could be cited, but our purpose is merely to document the fact that the negative binomial distribution has been used for years in widely divergent fields of application.

Here we are interested in the application of this particular distribution to the demand for aircraft spare parts. We shall first discuss the rationale which leads us to believe that it may offer a reasonable description of spare-parts demand.

Suppose that two-years' data on demand for spare parts is available at a base. For illustrative purposes, we will say that it is a base with four squadrons, each consisting of 18 aircraft. That is, the data covers the demands of 72 planes for 24 months. If all the aircraft were identical, of the same age, flew identical missions, underwent identical servicing, were subject to the same maintenance practices, etc., the demand for actuator parts might be expected to be about the same as in the illustrative example on page 4. In this case, the monthly demands would follow the Poisson distribution. If the population were homogeneous, random samples of demand data would be expected to exhibit only the sampling fluctuations inherent in any well-behaved variable. But the homogeneity

described above is not present in the Air Force environment in which the demands are generated. For example, even though the planes fly equal numbers of hours, make the same number of landings, etc., some planes are nevertheless subject to greater stresses than others because of factors of speed, altitude, rate of climb, etc. The probability of failure of a particular part is no longer constant, but has increased due to the stress factor. The recorded demands at the base for this part over the 24-month period might be as follows: 1-0-7-0-0-3-1-2-0-0-6-1-, 0-1-0-4-1-1-2-0-9-2-0-3. The sum of the demands at the base is 44. There are four squadrons, so that the demand rate per squadron month is the same as in the Poisson-fitted example on p. 4.

For these demand data, the Poisson is not a good fit. If we use the $\chi^2$ statistic to test the agreement of the observed distribution with the theoretical Poisson distribution, the computed $\chi^2$ value for 1 d.f. is 8.3. The probability of obtaining a $\chi^2$ value of this magnitude or greater in drawing random samples from a homogeneous population is less than .01. Such large values of chi-square may signify no more than the presence of an unusually divergent sample, but to the investigator who is none too sure of his hypothesis, the presence of repeated large chi-square values indicates that the hypothesis should be rejected.

However, if we use the negative binomial probability distribution with the same mean demand per month and assume that the ratio of variance to mean is 3, we will obtain a good fit for the sample data. ($\chi^2$ is .17 for 1 d.f., P = .70.) The discrepance between the observed frequency distribution of demands per month and the theoretical frequencies based on the negative binomial is very small indeed. The ratio of sample variance to sample mean in this case was about 3.2. A word of caution may be appropriate here concerning inferences made about the population on the basis of the chi-square test of goodness of fit. The viewpoint adopted is that the hypothesis is fixed—namely, that a negative binomial distribution describes the population of monthly demands for the part. The probability evaluated above is that of drawing from the population a sample more extreme than the one in hand. It is not a method for evaluating the probability that the hypothesis is correct. Of course, it is true that after the evidence from samples is accumulated the analyst-researcher must make a decision about the hypothesis, and his decision has some presumably high probability of being correct; but we have not presented a method of evaluating such a probability.

There are many factors in the Air Force environment that contribute to heterogeneity of demands. Among these factors we might list age of aircraft, applicability of parts, flying-program elements, nature of mission,

servicing schedules, maintenance practices, design change and modifications, changes in base personnel, etc. A crew mechanic inspecting plane number 1 may not have the same careful standards as the mechanic who inspects number 2. Some inspectors are prone to replace parts—some are not. Demands for certain spare parts have also been known to exhibit a tendency to occur in clusters—that is, the demand for a unit of part a stimulates demand for a unit of part b, which in turn creates a demand for 4 units of part c, etc. All of these factors change with time, and are reflected in the sample data.

## 4. TABLES OF NEGATIVE BINOMIAL DISTRIBUTION

We have found tables of the negative binomial distribution useful in our work in logistics. Since they may not be readily accessible, the complete probability distributions for a limited number of arbitrary paramater values are presented in the following pages. We have used 13 different values of the mean ($m = kp$) and 10 different ratios of variance-to-mean ($q$). This gives us 130 sets of parameter values for which the complete probability distribution is tabulated.

The distribution function of the negative binomial is

$$P(x) = \frac{(k+x-1)!}{(k-1)!\ x!} \frac{p^x}{q^{k+x}}$$

where $x = 0,1,2,3\cdots$, $p,k > 0$, and $q = 1 + p$.

Part 1 gives the individual terms of the distributions and Part 2 gives the cumulative probability of x demands or less. The values of the mean (kp) are .25(.25)1.0 and 1.0(1.0)10.0. The values of the ratio of variance to mean (q) are 1.5(.5)5.0 and 5.0(1.0)7.0. The choice of these values makes p vary from 1/2 to 6 and k from $\frac{1}{24}$ to 20.

## NEGATIVE BINOMIAL DISTRIBUTION PART I

Probability of x demands for q = 1.5

| x \ Mean | .25 | .50 | .75 | 1.0 | 2.0 | 3.0 | 4.0 | 5.0 | 6.0 | 7.0 | 8.0 | 9.0 | 10.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | .8165 | .6667 | .5443 | .4444 | .1975 | .0878 | .0390 | .0173 | .0077 | .0034 | .0015 | .0007 | .0003 |
| 1 | .1361 | .2222 | .2722 | .2963 | .2634 | .1756 | .1040 | .0578 | .0308 | .0160 | .0081 | .0041 | .0020 |
| 2 | .0340 | .0741 | .1134 | .1481 | .2195 | .2048 | .1561 | .1060 | .0668 | .0400 | .0230 | .0129 | .0070 |
| 3 | .0094 | .0247 | .0441 | .0658 | .1463 | .1821 | .1734 | .1413 | .1038 | .0710 | .0460 | .0286 | .0172 |
| 4 | .0028 | .0082 | .0165 | .0274 | .0854 | .1366 | .1590 | .1531 | .1299 | .1006 | .0729 | .0500 | .0328 |
| 5 | .0008 | .0027 | .0061 | .0110 | .0455 | .0910 | .1272 | .1429 | .1385 | .1206 | .0971 | .0733 | .0526 |
| 6 | .0003 | .0009 | .0022 | .0043 | .0228 | .0556 | .0918 | .1191 | .1308 | .1275 | .1133 | .0937 | .0731 |
| 7 | .0001 | .0003 | .0008 | .0016 | .0108 | .0318 | .0612 | .0907 | .1122 | .1214 | .1187 | .1071 | .0905 |
| 8 | | .0001 | .0003 | .0006 | .0050 | .0172 | .0383 | .0643 | .0888 | .1062 | .1138 | .1115 | .1018 |
| 9 | | | .0001 | .0002 | .0022 | .0089 | .0227 | .0428 | .0658 | .0866 | .1011 | .1074 | .1055 |
| 10 | | | | .0001 | .0010 | .0045 | .0129 | .0271 | .0460 | .0664 | .0843 | .0967 | .1020 |
| 11 | | | | | .0004 | .0022 | .0070 | .0164 | .0307 | .0483 | .0664 | .0820 | .0927 |
| 12 | | | | | .0002 | .0010 | .0037 | .0096 | .0196 | .0335 | .0498 | .0661 | .0799 |
| 13 | | | | | .0001 | .0005 | .0019 | .0054 | .0121 | .0223 | .0358 | .0508 | .0655 |
| 14 | | | | | | .0002 | .0009 | .0030 | .0072 | .0144 | .0247 | .0375 | .0515 |
| 15 | | | | | | .0001 | .0005 | .0016 | .0042 | .0089 | .0165 | .0267 | .0389 |
| 16 | | | | | | | .0002 | .0008 | .0023 | .0054 | .0106 | .0183 | .0284 |
| 17 | | | | | | | .0001 | .0004 | .0013 | .0032 | .0067 | .0122 | .0200 |
| 18 | | | | | | | | .0002 | .0007 | .0018 | .0041 | .0079 | .0137 |
| 19 | | | | | | | | .0001 | .0004 | .0010 | .0024 | .0050 | .0091 |
| 20 | | | | | | | | .0001 | .0002 | .0005 | .0014 | .0031 | .0059 |
| 21 | | | | | | | | | .0001 | .0003 | .0008 | .0019 | .0038 |
| 22 | | | | | | | | | | .0002 | .0005 | .0011 | .0023 |
| 23 | | | | | | | | | | .0001 | .0002 | .0006 | .0014 |
| 24 | | | | | | | | | | | .0001 | .0004 | .0009 |
| 25 | | | | | | | | | | | .0001 | .0002 | .0005 |

Probability of x demands for q = 2

| x \ Mean | .25 | .50 | .75 | 1.0 | 2.0 | 3.0 | 4.0 | 5.0 | 6.0 | 7.0 | 8.0 | 9.0 | 10.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | .8409 | .7071 | .5946 | .5000 | .2500 | .1250 | .0625 | .0312 | .0156 | .0078 | .0039 | .0020 | .0010 |
| 1 | .1051 | .1768 | .2230 | .2500 | .2500 | .1875 | .1250 | .0781 | .0459 | .0273 | .0156 | .0088 | .0049 |
| 2 | .0328 | .0663 | .0976 | .1250 | .1875 | .1875 | .1562 | .1172 | .0820 | .0547 | .0352 | .0220 | .0134 |
| 3 | .0123 | .0276 | .0447 | .0625 | .1250 | .1563 | .1562 | .1367 | .1094 | .0820 | .0586 | .0403 | .0269 |
| 4. | .0050 | .0121 | .0210 | .0312 | .0781 | .1172 | .1367 | .1367 | .1230 | .1025 | .0806 | .0604 | .0436 |
| 5 | .0021 | .0054 | .0100 | .0156 | .0469 | .0820 | .1094 | .1230 | .1230 | .1128 | .0967 | .0786 | .0611 |
| 6 | .0009 | .0025 | .0048 | .0078 | .0273 | .0547 | .0820 | .1025 | .1128 | .1128 | .1047 | .0916 | .0764 |
| 7 | .0004 | .0012 | .0023 | .0039 | .0156 | .0352 | .0586 | .0806 | .0967 | .1047 | .1047 | .0982 | .0873 |
| 8 | .0002 | .0005 | .0011 | .0020 | .0088 | .0220 | .0403 | .0604 | .0786 | .0916 | .0982 | .0982 | .0927 |
| 9 | .0001 | .0003 | .0005 | .0010 | .0049 | .0134 | .0269 | .0436 | .0611 | .0764 | .0873 | .0927 | .0927 |
| 10 | | .0001 | .0003 | .0005 | .0027 | .1081 | .0175 | .0305 | .0459 | .0611 | .0742 | .0835 | .0881 |
| 11 | | .0001 | .0001 | .0002 | .0015 | .0048 | .0111 | .0208 | .033[illegible] | .0472 | .0607 | .0721 | .0801 |
| 12 | | | .0001 | .0001 | .0008 | .0028 | .0069 | .0139 | .0236 | .0354 | .0481 | .0601 | .0701 |
| 13 | | | | .0001 | .0004 | .0016 | .0043 | .0091 | .0163 | .0259 | 0370 | .0485 | .0593 |
| 14 | | | | | .0002 | .0009 | .0026 | .0058 | .0111 | .0185 | .0277 | .0381 | .0487 |
| 15 | | | | | .0001 | .0005 | .0016 | .0037 | .0074 | .0129 | .0203 | .0292 | .0390 |
| 16 | | | | | .0001 | .0003 | .0009 | .0023 | .0049 | .0089 | .0146 | .0219 | .0304 |
| 17 | | | | | | .0002 | .0005 | .0014 | .0031 | .0060 | .0103 | .0161 | .0233 |
| 18 | | | | | | .0001 | .0003 | .0009 | .0020 | .0040 | .0072 | .0116 | .0175 |
| 19 | | | | | | .0001 | .0002 | .0005 | .0013 | .0026 | .0049 | .0082 | .0129 |
| 20 | | | | | | | .0001 | .0003 | .0008 | .0017 | .0033 | .0058 | .0093 |
| 21 | | | | | | | .0001 | .0002 | .0005 | .0011 | .0022 | .0040 | .0067 |
| 22 | | | | | | | | .0001 | .0003 | .0007 | .0015 | .0027 | .0047 |
| 23 | | | | | | | | .0001 | .0002 | .0004 | .0009 | .0018 | .0033 |
| 24 | | | | | | | | | .0001 | .0003 | .0006 | .0012 | .0022 |
| 25 | | | | | | | | | .0001 | .0002 | .0004 | .0008 | .0015 |

Probability of x demands for q = 2.5

| x \ Mean | .25 | .50 | .75 | 1.0 | 2.0 | 3.0 | 4.0 | 5.0 | 6.0 | 7.0 | 8.0 | 9.0 | 10.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | .8584 | .7368 | .6325 | .5429 | .2947 | .1600 | .0869 | .0472 | .0256 | .0139 | .0075 | .0041 | .0022 |
| 1 | .0858 | .1474 | .1897 | .2172 | .2358 | .1920 | .1390 | .0943 | .0614 | .0389 | .0241 | .0147 | .0089 |
| 2 | .0300 | .0589 | .0854 | .1086 | .1650 | .1728 | .1529 | .1226 | .0922 | .0662 | .0459 | .0310 | .0205 |
| 3 | .0130 | .0275 | .0427 | .0579 | .1100 | .1382 | .1427 | .1308 | .1106 | .0882 | .0673 | .0495 | .0355 |
| 4 | .0062 | .0138 | .0224 | .0318 | .0715 | .1037 | .1213 | .1242 | .1161 | .1014 | .0841 | .0669 | .0514 |
| 5 | .0031 | .0072 | .0121 | .0178 | .0458 | .0746 | .0970 | .1093 | .1115 | .1055 | .0941 | .0803 | .0658 |
| 6 | .0016 | .0038 | .0067 | .0101 | .0290 | .0523 | .0744 | .0911 | .1003 | .1020 | .0973 | .0883 | .0768 |
| 7 | .0008 | .0021 | .0037 | .0058 | .0182 | .0358 | .0553 | .0729 | .0860 | .0932 | .0946 | .0908 | .0834 |
| 8 | .0005 | .0011 | .0021 | .0033 | .0114 | .0242 | .0401 | .0565 | .0709 | .0816 | .0875 | .0885 | .0854 |
| 9 | .0002 | .0006 | .0012 | .0019 | .0071 | .0161 | .0285 | .0427 | .0568 | .0689 | .0777 | .0826 | .0835 |
| 10 | .0001 | .0004 | .0007 | .0011 | .0044 | .0106 | .0199 | .0316 | .0443 | .0565 | .0669 | .0744 | .0785 |
| 11 | .0001 | .0002 | .0004 | .0006 | .0027 | .0070 | .0138 | .0230 | .0338 | .0452 | .0559 | .0649 | .0714 |
| 12 | | .0001 | .0002 | .0004 | .0017 | .0045 | .0094 | .0165 | .0254 | .0354 | .0457 | .0552 | .0631 |
| 13 | | .0001 | .0001 | .0002 | .0010 | .0029 | .0064 | .0116 | .0187 | .0272 | .0365 | .0458 | .0543 |
| 14 | | | .0001 | .0001 | .0006 | .0019 | .0043 | .0081 | .0136 | .0206 | .0287 | .0373 | .0458 |
| 15 | | | | .0001 | .0004 | .0012 | .0029 | .0057 | .0098 | .0154 | .0222 | .0293 | .0379 |
| 16 | | | | | .0002 | .0008 | .0019 | .0039 | .0070 | .0114 | .0169 | .0235 | .0308 |
| 17 | | | | | .0001 | .0005 | .0012 | .0027 | .0049 | .0083 | .0127 | .0183 | .0246 |
| 18 | | | | | .0001 | .0003 | .0008 | .0018 | .0035 | .0060 | .0095 | .0140 | .0194 |
| 19 | | | | | .0001 | .0002 | .0005 | .0012 | .0024 | .0043 | .0070 | .0106 | .0151 |
| 20 | | | | | | .0001 | .0003 | .0008 | .0017 | .0030 | .0051 | .0080 | .0116 |
| 21 | | | | | | .0001 | .0002 | .0005 | .0011 | .0021 | .0037 | .0059 | .0089 |
| 22 | | | | | | | .0001 | .0004 | .0008 | .0015 | .0027 | .0044 | .0067 |
| 23 | | | | | | | .0001 | .0002 | .0005 | .0010 | .0019 | .0032 | .0050 |
| 24 | | | | | | | .0001 | .0002 | .0004 | .0007 | .0013 | .0023 | .0037 |
| 25 | | | | | | | | .0001 | .0002 | .0005 | .0009 | .0017 | .0027 |

Probability of x demands for q = 3.0

| x \ Mean | .25 | .50 | .75 | 1.0 | 2.0 | 3.0 | 4.0 | 5.0 | 6.0 | 7.0 | 8.0 | 9.0 | 10.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | .8717 | .7598 | .6623 | .5774 | .3333 | .1925 | .1111 | .0642 | .0370 | .0214 | .0123 | .0071 | .0041 |
| 1 | .0726 | .1266 | .1656 | .1925 | .2222 | .1925 | .1481 | .1069 | .0741 | .0499 | .0329 | .0214 | .0137 |
| 2 | .0272 | .0528 | .0759 | .0962 | .1481 | .1604 | .1481 | .1247 | .0988 | .0748 | .0549 | .0392 | .0274 |
| 3 | .0129 | .0264 | .0401 | .0535 | .0988 | .1247 | .1317 | .1247 | .1097 | .0915 | .0732 | .0566 | .0427 |
| 4 | .0067 | .0143 | .0225 | .0312 | .0658 | .0936 | .1097 | .1143 | .1097 | .0991 | .0854 | .0708 | .0569 |
| 5 | .0037 | .0081 | .0131 | .0187 | .0439 | .0686 | .0878 | .0991 | .1024 | .0991 | .0910 | .0802 | .0683 |
| 6 | .0021 | .0047 | .0078 | .0114 | .0293 | .0495 | .0683 | .0826 | .0910 | .0936 | .0910 | .0647 | .0759 |
| 7 | .0012 | .0028 | .0048 | .0071 | .0195 | .0354 | .0520 | .0669 | .0780 | .0847 | .0867 | .0647 | .0795 |
| 8 | .0007 | .0017 | .0029 | .0044 | .0130 | .0251 | .0390 | .0529 | .0650 | .0741 | .0795 | .0811 | .0795 |
| 9 | .0004 | .0010 | .0018 | .0028 | .0087 | .0176 | .0289 | .0412 | .0530 | .0631 | .0707 | .0751 | .0765 |
| 10 | .0003 | .0006 | .0011 | .0018 | .0058 | .0123 | .0212 | .0316 | .0424 | .0526 | .0612 | .0676 | .0714 |
| 11 | .0002 | .0004 | .0007 | .0011 | .0039 | .0086 | .0154 | .0239 | .0334 | .0430 | .0520 | .0594 | .0649 |
| 12 | .0001 | .0002 | .0005 | .0007 | .0026 | .0060 | .0111 | .0179 | .0260 | .0347 | .0433 | .0512 | .0577 |
| 13 | .0001 | .0002 | .0003 | .0005 | .0017 | .0041 | .0080 | .0133 | .0200 | .0276 | .0355 | .0433 | .0503 |
| 14 | | .0001 | .0002 | .0003 | .0011 | .0029 | .0057 | .0098 | .0152 | .0217 | .0287 | .0361 | .0431 |
| 15 | | .0001 | .0001 | .0002 | .0008 | .0020 | .0041 | .0072 | .0115 | .0168 | .0230 | .0297 | .0364 |
| 16 | | | .0001 | .0001 | .0005 | .0014 | .0029 | .0053 | .0086 | .0130 | .0182 | .0241 | .0304 |
| 17 | | | | .0001 | .0003 | .0009 | .0020 | .0038 | .0064 | .0099 | .0142 | .0194 | .0250 |
| 18 | | | | .0001 | .0002 | .0006 | .0014 | [illegible] | .0048 | .0075 | .0111 | .0154 | .0204 |
| 19 | | | | | .0002 | .0004 | .0010 | .0020 | .0035 | .0057 | .0086 | .0122 | .0164 |
| 20 | | | | | .0001 | .0003 | .0007 | .0014 | .0026 | .0043 | .0066 | .0095 | .0132 |
| 21 | | | | | .0001 | .0002 | .0005 | .0010 | .0019 | .0032 | .0050 | .0074 | .0104 |
| 22 | | | | | | .0001 | .0003 | .0007 | .0014 | .0024 | .0038 | .0057 | .0082 |
| 23 | | | | | | .0001 | .0002 | .0005 | .0010 | .0017 | .0029 | .0044 | .0064 |
| 24 | | | | | | .0001 | .0002 | .0004 | .0007 | .0013 | .0021 | .0034 | .0050 |
| 25 | | | | | | | .0001 | .0003 | .0005 | .0009 | .0016 | .0026 | .0039 |

Probability of x demands for q = 3.5

| x \ Mean | .25 | .50 | .75 | 1.0 | 2.0 | 3.0 | 4.0 | 5.0 | 6.0 | 7.0 | 8.0 | 9.0 | 10.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | .8823 | .7784 | .6867 | .6059 | .3671 | .2224 | .1347 | .0816 | .0495 | .0300 | .0182 | .0110 | .0067 |
| 1 | .0630 | .1112 | .1472 | .1731 | .2098 | .1906 | .1540 | .1166 | .0848 | .0599 | .0415 | .0283 | .0190 |
| 2 | .0248 | .0477 | .0683 | .0866 | .1348 | .1498 | .1430 | .1249 | .1030 | .0813 | .0622 | .0465 | .0340 |
| 3 | .0124 | .0296 | .0374 | .0495 | .0899 | .1141 | .1226 | .1190 | .1079 | .0930 | .0771 | .0620 | .0486 |
| 4 | .0069 | .0143 | .0220 | .0300 | .0610 | .0856 | .1007 | .1062 | .1040 | .0963 | .0853 | .0730 | .0607 |
| 5 | .0040 | .0086 | .0135 | .0189 | .0418 | .0636 | .0805 | .0911 | .0951 | .0935 | .0878 | .0793 | .0694 |
| 6 | .0024 | .0053 | .0085 | .0121 | .0289 | .0469 | .0633 | .0759 | .0838 | .0868 | .0857 | .0812 | .0743 |
| 7 | .0015 | .0034 | .0055 | .0079 | .0200 | .0345 | .0491 | .0620 | .0718 | .0780 | .0804 | .0795 | .0759 |
| 8 | .0010 | .0022 | .0036 | .0052 | .0140 | .0252 | .0377 | .0498 | .0603 | .0682 | .0732 | .0752 | .0745 |
| 9 | .0006 | .0014 | .0024 | .0035 | .0097 | .0184 | .0287 | .0395 | .0497 | .0585 | .0651 | .0693 | .0710 |
| 10 | .0004 | .0009 | .0016 | .0023 | .0068 | .0134 | .0217 | .0310 | .0405 | .0493 | .0567 | .0623 | .0659 |
| 11 | .0003 | .0006 | .0010 | .0016 | .0048 | .0098 | .0164 | .0242 | .0326 | .0410 | .0487 | .0551 | .0599 |
| 12 | .0002 | .0004 | .0007 | .0011 | .0034 | .0071 | .0123 | .0187 | .0260 | .0337 | .0411 | .0479 | .0535 |
| 13 | .0001 | .0003 | .0005 | .0007 | .0024 | .0051 | .0092 | .0144 | .0206 | .0274 | .0343 | .0410 | .0470 |
| 14 | .0001 | .0002 | .0003 | .0005 | .0017 | .0037 | .0068 | .0110 | .0162 | .0221 | .0284 | .0347 | .0408 |
| 15 | .0001 | .0001 | .0002 | .0003 | .0012 | .0027 | .0051 | .0084 | .0126 | .0176 | .0232 | .0291 | .0350 |
| 16 | | .0001 | .0002 | .0002 | .0008 | .0020 | .0038 | .0064 | .0098 | .0140 | .0189 | .0242 | .0296 |
| 17 | | .0001 | .0001 | .0002 | .0006 | .0014 | .0028 | .0048 | .0076 | .0111 | .0152 | .0199 | .0249 |
| 18 | | | .0001 | .0001 | .0004 | .0010 | .0021 | .0036 | .0058 | .0087 | .0122 | .0153 | .0208 |
| 19 | | | | .0001 | .0003 | .0007 | .0015 | .0027 | .0045 | .0068 | .0097 | .0132 | .0172 |
| 20 | | | | .0001 | .0002 | .0005 | .0011 | .0020 | .0034 | .0053 | .0077 | .0107 | .0145 |
| 21 | | | | | .0001 | .0004 | .0008 | .0015 | .0026 | .0041 | .0061 | .0086 | .0115 |
| 22 | | | | | .0001 | .0003 | .0006 | .0011 | .0020 | .0032 | .0048 | .0068 | .0093 |
| 23 | | | | | .0001 | .0002 | .0004 | .0009 | .0015 | .0024 | .0037 | .0054 | .0075 |
| 24 | | | | | .0001 | .0001 | .0003 | .0006 | .0011 | .0019 | .0029 | .0043 | .0061 |
| 25 | | | | | | .0001 | .0002 | .0005 | .0009 | .0014 | .0023 | .0034 | .0049 |

| x \ Mean | .25 | .50 | .75 | 1.0 | 2.0 | 3.0 | 4.0 | 5.0 | 6.0 | 7.0 | 8.0 | 9.0 | 10.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | .8909 | .7937 | .7071 | .6300 | .3969 | .2500 | .1575 | .0992 | .0625 | .0394 | .0248 | .0156 | .0098 |
| 1 | .0557 | .0992 | .1326 | .1575 | .1984 | .1875 | .1575 | .1240 | .0938 | .0689 | .0496 | .0352 | .0246 |
| 2 | .0226 | .0434 | .0621 | .0787 | .1240 | .1406 | .1378 | .1240 | .1055 | .0861 | .0682 | .0527 | .0400 |
| 3 | .0118 | .0235 | .0350 | .0459 | .0827 | .1055 | .1148 | .1137 | .1055 | .0933 | .0796 | .0659 | .0533 |
| 4 | .0068 | .0140 | .0213 | .0287 | .0568 | .0791 | .0933 | .0995 | .0989 | .0933 | .0846 | .0742 | .0633 |
| 5 | .0042 | .0087 | .0136 | .0187 | .0398 | .0593 | .0746 | .0846 | .0899 | .0866 | .0846 | .0779 | .0696 |
| 6 | .0027 | .0056 | .0089 | .0124 | .0282 | .0445 | .0591 | .0705 | .0779 | .0813 | .0810 | .0779 | .0725 |
| 7 | .0017 | .0037 | .0060 | .0084 | .0201 | .0334 | .0464 | .0579 | .0667 | .0725 | .0752 | .0751 | .0725 |
| 8 | .0011 | .0025 | .0041 | .0058 | .0145 | .0250 | .0363 | .0470 | .0563 | .0635 | .0682 | .0704 | .0703 |
| 9 | .0008 | .0017 | .0028 | .0040 | .0104 | .0188 | .0282 | .0379 | .0469 | .0547 | .0606 | .0645 | .0664 |
| 10 | .0005 | .0012 | .0019 | .0028 | .0076 | .0141 | .0219 | .0303 | .0387 | .0465 | .0530 | .0581 | .0614 |
| 11 | .0004 | .0008 | .0014 | .0020 | .0055 | .0106 | .0169 | .0241 | .0317 | .0391 | .0458 | .0515 | .0558 |
| 12 | .0003 | .0036 | .0010 | .0014 | .0040 | .0079 | .0130 | .0191 | .0257 | .0326 | .0391 | .0450 | .0500 |
| 13 | .0002 | .0004 | .0007 | .0010 | .0029 | .0059 | .0100 | .0150 | .0208 | .0269 | .0331 | .0390 | .0442 |
| 14 | .0001 | .0003 | .0005 | .0007 | .0021 | .0045 | .0077 | .0118 | .0167 | .0221 | .0278 | .0334 | .0387 |
| 15 | .0001 | .0002 | .0003 | .0005 | .0016 | .0033 | .0059 | .0093 | .0134 | .0181 | .0232 | .0284 | .0335 |
| 16 | .0001 | .0001 | .0002 | .0004 | .0012 | .0025 | .0045 | .0072 | .0106 | .0147 | .0192 | .0240 | .0288 |
| 17 |  | .0001 | .0002 | .0003 | .0009 | .0019 | .0035 | .0056 | .0085 | .0119 | .0158 | .0201 | .0246 |
| 18 |  | .0001 | .0001 | .0002 | .0006 | .0014 | .0026 | .0044 | .0067 | .0096 | .0129 | .0167 | .0208 |
| 19 |  | .0001 | .0001 | .0001 | .0005 | .0011 | .0020 | .0034 | .0053 | .0077 | .0106 | .0139 | .0175 |
| 20 |  |  | .0001 | .0001 | .0003 | .0008 | .0015 | .0026 | .0042 | .0061 | .0086 | .0114 | .0147 |
| 21 |  |  |  | .0001 | .0003 | .0006 | .0012 | .0020 | .0030 | .0049 | .0069 | .0094 | .0122 |
| 22 |  |  |  | .0001 | .0002 | .0004 | .0009 | .0016 | .0026 | .0039 | .0056 | .0077 | .0102 |
| 23 |  |  |  |  | .0001 | .0003 | .0007 | .0012 | .0020 | .0031 | .0045 | .0063 | .0084 |
| 24 |  |  |  |  | .0001 | .0003 | .0005 | .0009 | .0016 | .0024 | .0036 | .0051 | .0069 |
| 25 |  |  |  |  | .0001 | .0002 | .0004 | .0007 | .0012 | .0019 | .0029 | .0041 | .0057 |

Probability of x demands for q = 4.5

| x \ Mean | .25 | .50 | .75 | 1.0 | 2.0 | 3.0 | 4.0 | 5.0 | 6.0 | 7.0 | 8.0 | 9.0 | 10.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | .8981 | .8066 | .7245 | .6507 | .4234 | .2755 | .1793 | .1166 | .0759 | .0494 | .0321 | .0209 | .0136 |
| 1 | .0499 | .0896 | .1207 | .1446 | .1882 | .1837 | .1593 | .1296 | .1012 | .0768 | .0571 | .0418 | .0302 |
| 2 | .0208 | .0398 | .0570 | .0723 | .1150 | .1326 | .1328 | .1224 | .1068 | .0896 | .0730 | .0581 | .0453 |
| 3 | .0112 | .0221 | .0327 | .0428 | .0767 | .0983 | .1082 | .1088 | .1029 | .0929 | .0811 | .0688 | .0571 |
| 4 | .0067 | .0135 | .0205 | .0274 | .0532 | .0737 | .0872 | .0937 | .0943 | .0904 | .0834 | .0746 | .0650 |
| 5 | .0042 | .0087 | .0134 | .0182 | .0379 | .0557 | .0697 | .0791 | .0838 | .0843 | .0815 | .0762 | .0694 |
| 6 | .0028 | .0058 | .0091 | .0125 | .0273 | .0423 | .0555 | .0659 | .0729 | .0765 | .0770 | .0748 | .0707 |
| 7 | .0019 | .0040 | .0063 | .0087 | .0200 | .0322 | .0441 | .0544 | .0625 | .0680 | .0709 | .0713 | .0695 |
| 8 | .0013 | .0028 | .0044 | .0062 | .0147 | .0246 | .0349 | .0446 | .0530 | .0595 | .0640 | .0663 | .0666 |
| 9 | .0009 | .0019 | .0031 | .0044 | .0109 | .0189 | .0276 | .0363 | .0445 | .0514 | .0569 | .0606 | .0625 |
| 10 | .0006 | .0014 | .0022 | .0032 | .0081 | .0144 | .0217 | .0295 | .0371 | .0440 | .0499 | .0545 | .0577 |
| 11 | .0005 | .0010 | .0016 | .0023 | .0061 | .0111 | .0171 | .0238 | .0307 | .0373 | .0434 | .0485 | .0524 |
| 12 | .0003 | .0007 | .0012 | .0017 | .0045 | .0085 | .0135 | .0192 | .0253 | .0315 | .0373 | .0426 | .0471 |
| 13 | .0002 | .0005 | .0009 | .0013 | .0034 | .0066 | .0106 | .0154 | .0208 | .0264 | .0319 | .0372 | .0418 |
| 14 | .0002 | .0004 | .0006 | .0009 | .0026 | .0050 | .0083 | .0124 | .0170 | .0220 | .0271 | .0321 | .0369 |
| 15 | .0001 | .0003 | .0005 | .0007 | .0019 | .0039 | .0065 | .0099 | .0138 | .0182 | .0229 | .0276 | .0322 |
| 16 | .0001 | .0002 | .0003 | .0005 | .0015 | .0030 | .0051 | .0079 | .0112 | .0151 | .0192 | .0236 | .0280 |
| 17 | .0001 | .0002 | .0003 | .0004 | .0011 | .0023 | .0040 | .0063 | .0091 | .0124 | .0161 | .0200 | .0241 |
| 18 | | .0001 | .0002 | .0003 | .0008 | .0018 | .0032 | .0050 | .0074 | .0102 | .0134 | .0170 | .0207 |
| 19 | | .0001 | .0001 | .0002 | .0006 | .0014 | .0025 | .0040 | .0059 | .0083 | .011[illegible] | .0143 | .0177 |
| 20 | | .0001 | .0001 | .0002 | .0005 | .0011 | .0020 | .0032 | .0048 | .0068 | .00[illegible] | .0120 | .0150 |
| 21 | | | .0001 | .0001 | .0004 | .0008 | .0015 | .0025 | .0038 | .0055 | .0076 | .0100 | .0127 |
| 22 | | | | .0001 | .0003 | .0006 | .0012 | .0020 | .0031 | .0045 | .0063 | .0083 | .0107 |
| 23 | | | | | .0002 | .0005 | .0009 | .0016 | .0025 | .0037 | .0051 | .0069 | .0090 |
| 24 | | | | | .0002 | .0004 | .0007 | .0012 | .0020 | .0030 | .0042 | .0057 | .0076 |
| 25 | | | | | .0001 | .0003 | .0006 | .0010 | .0016 | .0024 | .0034 | .0047 | .0063 |

Probability of x demands for q = 5.0

| x \ Mean | .25 | .50 | .75 | 1.0 | 2.0 | 3.0 | 4.0 | 5.0 | 6.0 | 7.0 | 8.0 | 9.0 | 10.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | .9043 | .8178 | .7395 | .6687 | .4472 | .2991 | .2000 | .1337 | .0894 | .0598 | .0400 | .0267 | .0179 |
| 1 | .0452 | .0818 | .1109 | .1337 | .1789 | .1794 | .1600 | .1337 | .1073 | .0837 | .0640 | .0481 | .0358 |
| 2 | .0192 | .0368 | .0527 | .0669 | .1073 | .1256 | .1280 | .1204 | .1073 | .0921 | .0768 | .0626 | .0501 |
| 3 | .0106 | .0209 | .0307 | .0401 | .0716 | .0921 | .1024 | .1043 | .1002 | .0921 | .0819 | .0709 | .0601 |
| 4 | .0065 | .0130 | .0196 | .0261 | .0501 | .0691 | .0819 | .0887 | .0902 | .0875 | .0819 | .0745 | .0661 |
| 5 | .0042 | .0086 | .0131 | .0177 | .0361 | .0525 | .0655 | .0745 | .0793 | .0805 | .0786 | .0745 | .0688 |
| 6 | .0028 | .0059 | .0091 | .0124 | .0264 | .0402 | .0524 | .0621 | .0688 | .0725 | .0734 | .0720 | .0688 |
| 7 | .0020 | .0041 | .0064 | .0089 | .0196 | .0311 | .0419 | .0514 | .0589 | .0642 | .0671 | .0679 | .0668 |
| 8 | .0014 | .0029 | .0046 | .0064 | .0147 | .0241 | .0336 | .0424 | .0501 | .0562 | .0604 | .0628 | .0635 |
| 9 | .0010 | .0021 | .0034 | .0047 | .0111 | .0187 | .0268 | .0349 | .0423 | .0487 | .0537 | .0572 | .0592 |
| 10 | .0007 | .0015 | .0025 | .0035 | .0085 | .0146 | .0215 | .0286 | .0355 | .0419 | .0472 | .0515 | .0545 |
| 11 | .0005 | .0011 | .0018 | .0026 | .0065 | .0114 | .0172 | .0234 | .0297 | .0358 | .0412 | .0459 | .0495 |
| 12 | .0004 | .0008 | .0014 | .0020 | .0050 | .0089 | .0137 | .0191 | .0248 | .0304 | .0357 | .0405 | .0446 |
| 13 | .0003 | .0006 | .0010 | .0015 | .0038 | .0070 | .0110 | .0156 | .0206 | .0257 | .0308 | .0355 | .0398 |
| 14 | .0002 | .0005 | .0008 | .0011 | .0029 | .0055 | .0088 | .0127 | .0170 | .0217 | .0264 | .0310 | .0352 |
| 15 | .0002 | .0004 | .0006 | .0008 | .0023 | .0043 | .0070 | .0103 | .0141 | .0182 | .0225 | .0268 | .0310 |
| 16 | .0001 | .0003 | .0004 | .0006 | .0018 | .0034 | .0056 | .0084 | .0116 | .0153 | .0191 | .0231 | .0271 |
| 17 | .0001 | .0002 | .0004 | .0005 | .0014 | .0027 | .0045 | .0068 | .0096 | .0127 | .0162 | .0199 | .0236 |
| 18 | .0001 | .0002 | .0003 | .0004 | .0011 | .0021 | .0036 | .0055 | .0079 | .0106 | .0137 | .0170 | .0205 |
| 19 | .0001 | .0001 | .0003 | .0003 | .0008 | .0017 | .0029 | .0045 | .0065 | .0088 | .0115 | .0145 | .0177 |
| 20 | | .0001 | .0002 | .0002 | .0006 | .0013 | .0023 | .0036 | .0053 | .0073 | .0097 | .0123 | .0152 |
| 21 | | | .0002 | .0002 | .0005 | .0010 | .0018 | .0029 | .0043 | .0061 | .0081 | .0104 | .0130 |
| 22 | | | .0001 | .0001 | .0004 | .0008 | .0015 | .0024 | .0036 | .0050 | .0068 | .0088 | .0111 |
| 23 | | | .0001 | .0001 | .0003 | .0007 | .0012 | .0019 | .0029 | .0041 | .0057 | .0075 | .0095 |
| 24 | | | | .0001 | .0002 | .0005 | .0009 | .0016 | .0024 | .0034 | .0047 | .0063 | .0081 |
| 25 | | | | | .0002 | .0004 | .0008 | .0013 | .0019 | .0028 | .0039 | .0053 | .0068 |

Probability of x demands for q

| x \ Mean | .25 | .50 | .75 | 1.0 | 2.0 | 3.0 | 4.0 | 5.0 | 6.0 | 7.0 | 8.0 | 9.0 | 10.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | .9143 | .8360 | .7643 | .6988 | .4884 | .3413 | .2385 | .1667 | .1165 | .0814 | .0562 | .0397 | .0278 |
| 1 | .0381 | .0697 | .0955 | .1165 | .1628 | .1706 | .1590 | .1389 | .1165 | .0950 | .0758 | .0596 | .0463 |
| 2 | .0167 | .0319 | .0478 | .0582 | .0950 | .1138 | .1192 | .1157 | .1068 | .0950 | .0822 | .0696 | .0579 |
| 3 | .0095 | .0186 | .0273 | .0356 | .0633 | .0822 | .0927 | .0965 | .0949 | .0897 | .0822 | .0734 | .0643 |
| 4 | .0060 | .0120 | .0179 | .0237 | .0448 | .0616 | .0734 | .0804 | .0830 | .0822 | .0787 | .0734 | .0670 |
| 5 | .0041 | .0082 | .0124 | .0166 | .0329 | .0472 | .0587 | .0670 | .0720 | .0740 | .0735 | .0710 | .0670 |
| 6 | .0029 | .0058 | .0089 | .0120 | .0247 | .0367 | .0473 | .0558 | .0620 | .0658 | .0674 | .0670 | .0651 |
| 7 | .0021 | .0042 | .0065 | .0089 | .0188 | .0289 | .0383 | .0465 | .0531 | .0579 | .0609 | .0622 | .0620 |
| 8 | .0015 | .0031 | .0048 | .0066 | .0145 | .0229 | .0311 | .0388 | .0454 | .0507 | .0546 | .0571 | .0581 |
| 9 | .0011 | .0023 | .0037 | .0050 | .0113 | .0182 | .0253 | .0323 | .0387 | .0441 | .0485 | .0518 | .0538 |
| 10 | .0008 | .0018 | .0028 | .0039 | .0088 | .0146 | .0207 | .0269 | .0329 | .0382 | .0429 | .0466 | .0493 |
| 11 | .0006 | .0014 | .0021 | .0030 | .0070 | .0117 | .0169 | .0224 | .0279 | .0330 | .0377 | .0417 | .0449 |
| 12 | .0005 | .0010 | .0017 | .0023 | .0055 | .0094 | .0139 | .0187 | .0236 | .0284 | .0330 | .0370 | .0405 |
| 13 | .0004 | .0008 | .0013 | .0018 | .0044 | .0076 | .0114 | .0156 | .0200 | .0244 | .0287 | .0328 | .0363 |
| 14 | .0003 | .0006 | .0010 | .0014 | .0035 | .0062 | .0094 | .0130 | .0169 | .0209 | .0250 | .0289 | .0325 |
| 15 | .0002 | .0005 | .0008 | .0011 | .0028 | .0050 | .0077 | .0108 | .0143 | .0179 | .0216 | .0253 | .0288 |
| 16 | .0002 | .0004 | .0006 | .0009 | .0022 | .0041 | .0063 | .0090 | .0120 | .0153 | .0187 | .0222 | .0255 |
| 17 | .0001 | .0003 | .0005 | .0007 | .0018 | .0033 | .0052 | .0075 | .0101 | .0130 | .0161 | .0193 | .0225 |
| 18 | .0001 | .0002 | .0004 | .0006 | .0015 | .0027 | .0043 | .0063 | .0085 | .0111 | .0139 | .0168 | .0198 |
| 19 | .0001 | .0002 | .0003 | .0004 | .0012 | .0022 | .0035 | .0052 | .0072 | .0095 | .0120 | .0146 | .0174 |
| 20 | .0001 | .0002 | .0003 | .0004 | .0010 | .0018 | .0029 | .0043 | .0061 | .0080 | .0103 | .0127 | .0152 |
| 21 | .0001 | .0001 | .0002 | .0003 | .0008 | .0015 | .0024 | .0036 | .0051 | .0068 | .0088 | .0110 | .0133 |
| 22 |  | .0001 | .0002 | .0002 | .0006 | .0012 | .0020 | .0030 | .0043 | .0058 | .0075 | .0095 | .0116 |
| 23 |  | .0001 | .0001 | .0002 | .0005 | .0010 | .0016 | .0025 | .0036 | .0049 | .0064 | .0082 | .0101 |
| 24 |  | .0001 | .0001 | .0002 | .0004 | .0008 | .0014 | .0021 | .0030 | .0042 | .0055 | .0070 | .0087 |
| 25 |  | .0001 | .0001 | .0001 | .0003 | .0007 | .0011 | .0017 | .0025 | .0035 | .0047 | .0060 | .0076 |

Cumulative probability $\sum_{0}^{x} P$ for q = 2.0

| x \ Mean | .25 | .50 | .75 | 1.0 | 2.0 | 3.0 | 4.0 | 5.0 | 6.0 | 7.0 | 8.0 | 9.0 | 10.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | .8409 | .7071 | .5946 | .5000 | .2500 | .1250 | .0625 | .0312 | .0156 | .0078 | .0039 | .0020 | .0010 |
| 1 | .9460 | .8839 | .8176 | .7500 | .5000 | .3125 | .1875 | .1094 | .0625 | .0352 | .0195 | .0107 | .0059 |
| 2 | .9789 | .9502 | .9151 | .8750 | .6875 | .5000 | .3438 | .2266 | .1445 | .0898 | .0547 | .0327 | .0193 |
| 3 | .9912 | .9778 | .9598 | .9375 | .8125 | .6562 | .5000 | .3633 | .2539 | .1719 | .1133 | .0730 | .0461 |
| 4 | .9962 | .9899 | .9808 | .9688 | .8906 | .7734 | .6367 | .5000 | .3770 | .2744 | .1938 | .1334 | .0898 |
| 5 | .9983 | .9953 | .9908 | .9844 | .9375 | .8555 | .7471 | .6230 | .5000 | .3872 | .2905 | .2120 | .1509 |
| 6 | .9992 | .9978 | .9955 | .9922 | .9648 | .9102 | .8281 | .7256 | .6128 | .5000 | .3953 | .3036 | .2272 |
| 7 | .9997 | .9989 | .9978 | .9961 | .9805 | .9453 | .8867 | .8062 | .7095 | .6047 | .5000 | .4018 | .3145 |
| 8 | .9998 | .9995 | .9989 | .9980 | .9893 | .9673 | .9270 | .8666 | .7880 | .6964 | .5982 | .5000 | .4073 |
| 9 | .9999 | .9998 | .9995 | .9990 | .9941 | .9807 | .9539 | .9102 | .8491 | .7728 | .6855 | .5927 | .5000 |
| 10 | | .9999 | .9997 | .9995 | .9968 | .9888 | .9713 | .9408 | .8949 | .8338 | .7597 | .6762 | .5881 |
| 11 | | | .9999 | .9998 | .9983 | .9935 | .9824 | .9616 | .9283 | .8811 | .8204 | .7483 | .6682 |
| 12 | | | | .9999 | .9991 | .9963 | .9894 | .9755 | .9519 | .9165 | .8684 | .8083 | .7333 |
| 14 | | | | | .9995 | .9979 | .9936 | .9846 | .9682 | .9423 | .9054 | .8569 | .7976 |
| 15 | | | | | .9997 | .9988 | .9962 | .9904 | .9793 | .9608 | .9331 | .8950 | .8463 |
| 16 | | | | | .9999 | .9993 | .9977 | .9941 | .9867 | .9738 | .9534 | .9242 | .8852 |
| 17 | | | | | | .9996 | .9987 | .9964 | .9915 | .9827 | .9680 | .9461 | .9157 |
| 18 | | | | | | .9998 | .9992 | .9978 | .9947 | .9837 | .9784 | .9622 | .9390 |
| 19 | | | | | | .9999 | .9996 | .9987 | .9967 | .9927 | .9855 | .9739 | .9564 |
| 20 | | | | | | | .9998 | .9992 | .9980 | .9953 | .9904 | .9822 | .9693 |
| 21 | | | | | | | .9999 | .9995 | .9988 | .9970 | .9937 | .9879 | .9786 |
| 22 | | | | | | | | .9997 | .9992 | .9981 | .9959 | .9919 | .9853 |
| 23 | | | | | | | | .9998 | .9995 | .9988 | .9974 | .9947 | .9900 |
| 24 | | | | | | | | .9999 | .9997 | .9993 | .9983 | .9965 | .9932 |
| 25 | | | | | | | | | .9998 | .9996 | .9989 | .9977 | .9955 |
| | | | | | | | | | .9999 | .9997 | .9993 | .9985 | [illegible] |

Cumulative probability $\sum_{0}^{x} P$ for $q = 2.5$

| x \ Mean | .25 | .50 | .75 | 1.0 | 2.0 | 3.0 | 4.0 | 5.0 | 6.0 | 7.0 | 8.0 | 9.0 | 10.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | .8584 | .7368 | .6325 | .5429 | .2947 | .1600 | .0869 | .0472 | 0256 | .0139 | .0075 | .0041 | .0022 |
| 1 | .9442 | .8842 | .8222 | .7600 | .5305 | .3520 | .2258 | .1415 | .0870 | .0528 | .0317 | .0188 | .0111 |
| 2 | .9743 | .9431 | .9076 | .8686 | .6955 | .5248 | .3787 | .2641 | .1792 | .1190 | .0776 | .0498 | .0316 |
| 3 | .9873 | .9706 | .9503 | .9265 | .8056 | .6530 | .5214 | .3948 | .2898 | .2072 | .1448 | .0994 | .0670 |
| 4 | .9935 | .9844 | .9727 | .9584 | .8771 | .7667 | .6427 | .5191 | .4059 | .3086 | .2289 | .1662 | .1185 |
| 5 | .9965 | .9915 | .9848 | .9762 | .9229 | .8414 | .7397 | .6284 | .5174 | .4141 | .3231 | .2465 | .1843 |
| 6 | .9981 | .9953 | .9914 | .9863 | .9519 | .8936 | .8141 | .7195 | .6177 | .5161 | .4205 | .3348 | .2610 |
| 7 | .9990 | .9974 | .9951 | .9921 | .9701 | .9295 | .8694 | .7924 | .7037 | .6093 | .5150 | .4256 | .3444 |
| 8 | .9994 | .9985 | .9972 | .9954 | .9815 | .9536 | .9094 | .8489 | .7747 | .6909 | .6025 | .5141 | .4299 |
| 9 | .9997 | .9992 | .9984 | .9973 | .9886 | .9698 | .9379 | .8916 | .8314 | .7598 | .6802 | .5968 | .5134 |
| 10 | .9998 | .9995 | .9991 | .9984 | .9929 | .9804 | .9578 | .9231 | .8757 | .8163 | .7471 | .6712 | .5919 |
| 11 | .9999 | .9997 | .9995 | .9991 | .9957 | .9874 | .9716 | .9461 | .9095 | .8615 | .8030 | .7361 | .6633 |
| 12 | | .9998 | .9997 | .9995 | .9973 | .9919 | .9810 | .9626 | .9349 | .8969 | .8487 | .7912 | .7264 |
| 13 | | .9999 | .9998 | .9997 | .9984 | .9948 | .9874 | .9742 | .9536 | .9241 | .8852 | .8371 | .7807 |
| 14 | | | .9999 | .9998 | .9990 | .9967 | .9917 | .9824 | .9672 | .9447 | .9139 | .8744 | .8265 |
| 15 | | | | .9999 | .9994 | .9979 | .9945 | .9881 | .9770 | .9601 | .9361 | .9043 | .8644 |
| 16 | | | | | .9996 | .9987 | .9964 | .9919 | .9840 | .9715 | .9530 | .9278 | .8952 |
| 17 | | | | | .9998 | .9992 | .9977 | .9946 | .9890 | .9797 | .9658 | .9460 | .9198 |
| 18 | | | | | .9999 | .9995 | .9985 | .9964 | .9924 | .9857 | .9753 | .9600 | .9392 |
| 19 | | | | | | .9997 | .9990 | .9976 | .9948 | .9900 | .9822 | .9706 | .9543 |
| 20 | | | | | | .9998 | .9994 | .9984 | .9965 | .9930 | .9874 | .9786 | .9659 |
| 21 | | | | | | .9999 | .9996 | .9990 | .9976 | .9952 | .9910 | .9845 | .9748 |
| 22 | | | | | | | .9997 | .9993 | .9984 | .9967 | .9937 | .9889 | .9815 |
| 23 | | | | | | | .9998 | .9996 | .9989 | .9977 | .9956 | .9920 | .9865 |
| 24 | | | | | | | .9999 | .9997 | .9993 | .9985 | .9969 | .9943 | .9902 |
| 25 | | | | | | | | 9998 | .9995 | .9990 | .9979 | .9960 | .9930 |

Cumulative probability $\sum_{0}^{x} P$ for q = 3.0

| x \ Mean | .25 | .50 | .75 | 1.0 | 2.0 | 3.0 | 4.0 | 5.0 | 6.0 | 7.0 | 8.0 | 9.0 | 10.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | .8717 | .7598 | .6623 | .5774 | .3333 | .1925 | .1111 | .0642 | .0370 | .0214 | .0123 | .0071 | .0041 |
| 1 | .9443 | .8865 | .8279 | .7698 | .5556 | .3849 | .2593 | .1711 | .1111 | .0713 | .0453 | .0285 | .0178 |
| 2 | .9716 | .9392 | .9038 | .8660 | .7037 | .5453 | .4074 | .2958 | .2099 | .1461 | .1001 | .0677 | .0453 |
| 3 | .9844 | .9656 | .9439 | .9195 | .8025 | .6700 | .5391 | .4205 | .3196 | .2376 | .1733 | .1243 | .0879 |
| 4 | .9911 | .9799 | .9664 | .9407 | .8683 | .7636 | .6488 | .5349 | .4294 | .3367 | .2586 | .1951 | .1448 |
| 5 | .9948 | .9880 | .9795 | .9694 | .9122 | .8322 | .7366 | .6340 | .5318 | .4358 | .3497 | .2753 | .2131 |
| 6 | .9969 | .9927 | .9874 | .9808 | .9415 | .8817 | .8049 | .7166 | .6228 | .5294 | .4407 | .3600 | .2890 |
| 7 | .9981 | .9955 | .9922 | .9879 | .9610 | .9171 | .8569 | .7834 | .7009 | .6141 | .5274 | .4447 | .3685 |
| 8 | .9989 | .9972 | .9951 | .9923 | .9740 | .9422 | .8960 | .8363 | .7659 | .6881 | .6069 | .5258 | .4480 |
| 9 | .9993 | .9983 | .9969 | .9951 | .9827 | .9598 | .9249 | .8775 | .8189 | .7513 | .6776 | .6010 | .5245 |
| 10 | .9996 | .9989 | .9980 | .9969 | .9884 | .9722 | .9460 | .9091 | .8613 | .8039 | .7388 | .6686 | .5959 |
| 11 | .9997 | .9993 | .9988 | .9980 | .9923 | .9808 | .9615 | .9330 | .8947 | .8469 | .7908 | .7280 | .6609 |
| 12 | .9998 | .9996 | .9992 | .9987 | .9949 | .9868 | .9726 | .9509 | .9206 | .8816 | .8341 | .7792 | .7186 |
| 13 | .9999 | .9997 | .9995 | .9992 | .9966 | .9909 | .9806 | .9642 | .9406 | .9091 | .8696 | .8225 | .7689 |
| 14 | | .9998 | .9997 | .9995 | .9977 | .9937 | .9863 | .9741 | .9558 | .9308 | .8983 | .8586 | .8121 |
| 15 | | .9999 | .9998 | .9996 | .9985 | .9957 | .9904 | .9813 | .9674 | .9476 | .9213 | .8883 | .8485 |
| 16 | | | .9999 | .9998 | .9990 | .9971 | .9932 | .9865 | .9760 | .9606 | .9396 | .9124 | .8788 |
| 17 | | | | .9999 | .9993 | .9980 | .9953 | .9904 | .9824 | .9705 | .9538 | .9317 | .9038 |
| 18 | | | | | .9995 | .9986 | .9967 | .9931 | .9872 | .9780 | .9649 | .9472 | .9242 |
| 19 | | | | | .9997 | .9991 | .9977 | .9951 | .9907 | .9837 | .9735 | .9594 | .9406 |
| 20 | | | | | .9998 | .9994 | .9984 | .9965 | .9935 | .9880 | .9801 | .9689 | .9538 |
| 21 | | | | | .9999 | .9996 | .9989 | .9975 | .9951 | .9912 | .9851 | .9763 | .9642 |
| 22 | | | | | | .9997 | .9992 | .9983 | .9965 | .9935 | .9889 | .9821 | .9725 |
| 23 | | | | | | .9998 | .999 | .9988 | .9975 | .9953 | .9918 | .9865 | .9789 |
| 24 | | | | | | .9999 | .9996 | .9991 | .9982 | .9966 | .9939 | .9898 | .9839 |
| 25 | | | | | | | .9997 | .9994 | .987 | [illegible] | .9955 | .9924 | [illegible] |

Cumulative probability $\sum_{0}^{x} P$ for q = 3.5

| x \ Mean | .25 | .50 | .75 | 1.0 | 2.0 | 3.0 | 4.0 | 5.0 | 6.0 | 7.0 | 8.0 | 9.0 | 10.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | .8823 | .7784 | .6867 | .6059 | .3671 | .2224 | .1347 | .0816 | .0495 | .0300 | .0182 | .0110 | .0067 |
| 1 | .9453 | .8896 | .8339 | .7790 | .5768 | .4130 | .2887 | .1983 | .1342 | .0899 | .0597 | .0393 | .0257 |
| 2 | .9700 | .9372 | .9022 | .8655 | .7117 | .5628 | .4317 | .3232 | .2372 | .1712 | .1219 | .0857 | .0597 |
| 3 | .9824 | .9622 | .9396 | .9150 | .8016 | .6769 | .5543 | .4422 | .3451 | .2642 | .1990 | .1477 | .1083 |
| 4 | .9893 | .9764 | .9617 | .9450 | .8626 | .7625 | .6549 | .5484 | .4491 | .3604 | .2843 | .2207 | .1690 |
| 5 | .9933 | .9850 | .9752 | .9639 | .9044 | .8261 | .7355 | .6395 | .5441 | .4540 | .3720 | .3000 | .2384 |
| 6 | .9957 | .9903 | .9837 | .9760 | .9333 | .8730 | .7988 | .7154 | .6279 | .5408 | .4577 | .3812 | .3127 |
| 7 | .9972 | .9937 | .9892 | .9839 | .9533 | .9075 | .8478 | .7774 | .6997 | .6188 | .5381 | .4607 | .3885 |
| 8 | .9982 | .9958 | .9928 | .9892 | .9673 | .9327 | .8855 | .8271 | .7600 | .6870 | .6114 | .5359 | .4631 |
| 9 | .9988 | .9972 | .9952 | .9927 | .9770 | .9511 | .9142 | .8667 | .8097 | .7455 | .6765 | .6052 | .5340 |
| 10 | .9992 | .9981 | .9967 | .9950 | .9838 | .9646 | .9360 | .8977 | .8502 | .7948 | .7332 | .6675 | .5999 |
| 11 | .9995 | .9987 | .9978 | .9966 | .9886 | .9743 | .9524 | .9219 | .8828 | .8358 | .7818 | .7226 | .6598 |
| 12 | .9996 | .9992 | .9985 | .9977 | .9920 | .9814 | .9646 | .9406 | .9088 | .8694 | .8229 | .7704 | .7133 |
| 13 | .9998 | .9994 | .9990 | .9984 | .9943 | .9866 | .9738 | .9550 | .9294 | .8968 | .8573 | .8115 | .7603 |
| 14 | .9998 | .9996 | .9993 | .9989 | .9960 | .9903 | .9806 | .9660 | .9456 | .9189 | .8856 | .8462 | .8011 |
| 15 | .9999 | .9997 | .9995 | .9992 | .9972 | .9930 | .9857 | .9744 | .9582 | .9365 | .9089 | .8753 | .8361 |
| 16 | | .9998 | .9997 | .9995 | .9980 | .9949 | .9895 | .9808 | .9680 | .9505 | .9278 | .8995 | .8657 |
| 17 | | .9999 | .9998 | .9996 | .9986 | .9964 | .9923 | .9856 | .9756 | .9616 | .9430 | .9194 | .8906 |
| 18 | | | .9998 | .9997 | .9990 | .9973 | .9943 | .9892 | .9815 | .9703 | .9552 | .9357 | .9114 |
| 19 | | | .9999 | .9998 | .9993 | .9981 | .9958 | .9920 | .9859 | .9771 | .9649 | .9489 | .9285 |
| 20 | | | | .9999 | .9995 | .9986 | .9969 | .9940 | .9894 | .9824 | .9727 | .9595 | .9427 |
| 21 | | | | | .9996 | .9990 | .9978 | .9956 | .9920 | .9865 | .9787 | .9681 | .9542 |
| 22 | | | | | .9997 | .9993 | .9984 | .9967 | .9940 | .9897 | .9835 | .9749 | .9635 |
| 23 | | | | | .9998 | .9995 | .9988 | .9976 | .9955 | .9921 | .9873 | .9804 | .9710 |
| 24 | | | | | .9999 | .9996 | .9991 | .9982 | .9966 | .9940 | .9902 | .9847 | .9771 |
| 25 | | | | | | .9997 | .9994 | 9987 | .9974 | .9955 | .9925 | .9881 | .9820 |

Cumulative probability $\sum_{0}^{x} P$ for $q = 4.0$

| x \ Mean | .25 | .50 | .75 | 1.0 | 2.0 | 3.0 | 4.0 | 5.0 | 6.0 | 7.0 | 8.0 | 9.0 | 10.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | .8909 | .7937 | .7071 | .6300 | .3969 | .2500 | .1575 | .0992 | .0625 | .0394 | .0248 | .0156 | .0098 |
| 1 | .9466 | .8929 | .8397 | .7875 | .5953 | .4375 | .3150 | .2232 | .1562 | .1083 | .0744 | .0508 | .0344 |
| 2 | .9692 | .9363 | .9018 | .8662 | .7193 | .5781 | .4528 | .3472 | .2617 | .1944 | .1426 | .1035 | .0744 |
| 3 | .9810 | .9598 | .9368 | .9121 | .8020 | .6836 | .5676 | .4609 | .3672 | .2877 | .2222 | .1694 | .1278 |
| 4 | .9878 | .9738 | .9581 | .9408 | .8588 | .7627 | .6609 | .5604 | .4661 | .3810 | .3067 | .2436 | .1911 |
| 5 | .9920 | .9825 | .9717 | .9595 | .8986 | .8220 | .7356 | .6449 | .5551 | .4697 | .3913 | .3215 | .2607 |
| 6 | .9946 | .9881 | .9806 | .9719 | .9268 | .8665 | .7947 | .7154 | .6329 | .5509 | .4723 | .3993 | .3333 |
| 7 | .9963 | .9919 | .9866 | .9804 | .9469 | .8999 | .8411 | .7733 | .6997 | .62[illegible]5 | .5476 | .4744 | .4058 |
| 8 | .9975 | .9944 | .9906 | .9862 | .9614 | .9249 | .8774 | .8203 | .7560 | .68[illegible]9 | .6157 | .5447 | .4761 |
| 9 | .9983 | .9961 | .9934 | .9902 | .9718 | .9437 | .9056 | .8582 | .8029 | .7416 | .6764 | .6093 | .5425 |
| 10 | .9988 | .9972 | .9953 | .9930 | .9794 | .9578 | .9274 | .8885 | .8416 | .7881 | .7294 | .6674 | .6039 |
| 11 | .9992 | .9981 | .9967 | .9950 | .9849 | .9683 | .9443 | .9126 | .8733 | .8271 | .7752 | .7189 | .6597 |
| 12 | .9994 | .9986 | .9976 | .9964 | .9889 | .9762 | .9574 | .9317 | .8990 | .8597 | .8143 | .7639 | .7097 |
| 13 | .9996 | .9990 | .9983 | .9974 | .9919 | .9822 | .9674 | .9467 | .9198 | .8866 | .8474 | .8029 | .7539 |
| 14 | .9997 | .9993 | .9988 | .9982 | .9940 | .9866 | .9751 | .9586 | .9365 | .9087 | .8752 | .8363 | .7926 |
| 15 | .9998 | .9995 | .9991 | .9987 | .9956 | .9900 | .9810 | .9678 | .9499 | .9268 | .8984 | .8647 | .8262 |
| 16 | .9998 | .9996 | .9994 | .9990 | .9968 | .9925 | .9855 | .9750 | .9605 | .9415 | .9175 | .8887 | .8550 |
| 17 | .9999 | .9997 | .9995 | .9993 | .9976 | .9944 | .9889 | .9807 | .9690 | .9533 | .9333 | .9087 | .8796 |
| 18 | | .9998 | .9997 | .9995 | .9982 | .9958 | .9916 | .9851 | .9757 | .9629 | .9463 | .9255 | .9004 |
| 19 | | .9999 | .9998 | .9996 | .9987 | .9968 | .9936 | .9885 | .9810 | .9706 | .9568 | .9394 | .9179 |
| 20 | | | .9998 | .9997 | .9990 | .9976 | .9951 | .9911 | .9851 | .9767 | .9654 | .9508 | .9326 |
| 21 | | | .9999 | .9998 | .9993 | .9982 | .9963 | .9932 | .9884 | .9816 | .9723 | .9602 | .9449 |
| 22 | | | | .9999 | .9995 | .9987 | .9972 | .9947 | .9910 | .9855 | .9779 | .9679 | .9550 |
| 23 | | | | | .9996 | .9990 | .9979 | .9960 | .9930 | .9886 | .9824 | .9742 | .9634 |
| 24 | | | | | .9997 | .9992 | .9984 | .9969 | .9945 | .9910 | .9861 | .9793 | .9703 |
| 25 | | | | | .9998 | .99[illegible]4 | .9988 | .9976 | .[illegible]58 | .9930 | .9889 | .9834 | .9760 |

Cumulative probability $\sum_{0}^{x} P$ for q = 4.5

| x \ Mean | .25 | .50 | .75 | 1.0 | 2.0 | 3.0 | 4.0 | 5.0 | 6.0 | 7.0 | 8.0 | 9.0 | 10.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | .8981 | .8066 | .7245 | .6507 | .4234 | .2755 | .1793 | .1166 | .0759 | .0494 | .0321 | .0209 | .0136 |
| 1 | .9480 | .8963 | .8452 | .7953 | .6116 | .4591 | .3386 | .2462 | .1771 | .1262 | .0893 | .0627 | .0438 |
| 2 | .9688 | .9361 | .9022 | .8676 | .7265 | .5918 | .4714 | .3686 | .2839 | .2158 | .1622 | .1208 | .0892 |
| 3 | .9800 | .9582 | .9350 | .9104 | .8032 | .6900 | .5796 | .4774 | .3868 | .3088 | .2434 | .1896 | .1463 |
| 4 | .9867 | .9718 | .9554 | .9378 | .8564 | .7637 | .6667 | .5711 | .4810 | .3991 | .3267 | .2642 | .2113 |
| 5 | .9909 | .9805 | .9688 | .9560 | .8943 | .8194 | .7364 | .6502 | .5649 | .4835 | .4082 | .3404 | .2807 |
| 6 | .9937 | .9863 | .9779 | .9685 | .9216 | .8617 | .7920 | .7162 | .6378 | .5600 | .4852 | .4152 | .3514 |
| 7 | .9955 | .9903 | .9842 | .9773 | .9416 | .8939 | .8360 | .7706 | .7003 | .6280 | .5560 | .4865 | .4209 |
| 8 | .9968 | .9930 | .9886 | .9835 | .9563 | .9185 | .8709 | .8152 | .7533 | .6875 | .6200 | .5528 | .4875 |
| 9 | .9977 | .9949 | .9917 | .9879 | .9672 | .9373 | .8985 | .8515 | .7978 | .7390 | .6769 | .6134 | .5500 |
| 10 | .9983 | .9963 | .9939 | .9911 | .9753 | .9518 | .9202 | .8810 | .8348 | .7830 | .7268 | .6679 | .6077 |
| 11 | .9988 | .9973 | .9955 | .9934 | .9814 | .9629 | .9374 | .9048 | .8655 | .8203 | .7702 | .7163 | .6601 |
| 12 | .9991 | .9980 | .9967 | .9951 | .9859 | .9714 | .9508 | .9240 | .8908 | .8518 | .8075 | .7590 | .7072 |
| 13 | .9993 | .9985 | .9976 | .9964 | .9893 | .9779 | .9614 | .9394 | .9116 | .8781 | .8394 | .7961 | .7491 |
| 14 | .9995 | .9989 | .9982 | .9973 | .9919 | .9830 | .9698 | .9517 | .9285 | .9001 | .8665 | .8283 | .7859 |
| 15 | .9996 | .9992 | .9986 | .9980 | .9938 | .9869 | .9763 | .9616 | .9424 | .9183 | .8894 | .8559 | .8182 |
| 16 | .9997 | .9994 | .9990 | .9985 | .9953 | .9898 | .9814 | .9695 | .9536 | .9334 | .9086 | .8795 | .8461 |
| 17 | .9998 | .9996 | .9992 | .9989 | .9964 | .9922 | .9855 | .9758 | .9627 | .9458 | .9247 | .8995 | .8703 |
| 18 | .9999 | .9997 | .9994 | .9991 | .9973 | .9939 | .9886 | .9808 | .9701 | .9559 | .9381 | .9165 | .8910 |
| 19 | | .9998 | .9996 | .9994 | .9979 | .9953 | .9911 | .9848 | .9760 | .9643 | .9493 | .9308 | .9086 |
| 20 | | .9998 | .9997 | .9995 | .9984 | .9964 | .9930 | .9880 | .9808 | .9711 | .9585 | .9428 | .9237 |
| 21 | | .9999 | .9998 | .9996 | .9988 | .9972 | .9946 | .9905 | .9846 | .9766 | .9661 | .9528 | .9364 |
| 22 | | | .9998 | .9997 | .9991 | .9978 | .9957 | .9925 | .9877 | .9811 | .9724 | .9611 | .9471 |
| 23 | | | .9999 | .9998 | .9993 | .9983 | .9967 | .9941 | .9902 | .9848 | .9775 | .9680 | .9561 |
| 24 | | | | .9998 | .9995 | .9987 | .9974 | .9953 | .9922 | .9878 | .9817 | .9738 | .9637 |
| 25 | | | | .9999 | .9996 | .9990 | .9980 | .9963 | .9938 | .9902 | .9852 | .9785 | .9700 |

[illegible]mulative probability [illegible] for $q_0$ [illegible]

| r \ Mean | .25 | .50 | .75 | 1.0 | 2.0 | 3.0 | 4.0 | 5.0 | 6.0 | 7.0 | 8.0 | 9.0 | 10.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | .9043 | .8178 | .7395 | .6687 | .4472 | .2991 | .2000 | .1337 | .0894 | .0598 | .0400 | .0267 | .0179 |
| 1 | .9495 | .8995 | .8504 | .8025 | .6261 | .4785 | .3600 | .2675 | .1968 | .1436 | .1040 | .0749 | .0537 |
| 2 | .9687 | .9363 | .9031 | .8694 | .7334 | .6041 | .4880 | .3879 | .3041 | .2357 | .1808 | .1375 | .1038 |
| 3 | .9793 | .9572 | .9339 | .9095 | .8050 | .6962 | .5904 | .4922 | .4043 | .3278 | .2627 | .2084 | .1639 |
| 4 | .9858 | .9702 | .9535 | .9356 | .8551 | .7653 | .6723 | .5809 | .4[illegible]44 | .4153 | .3446 | .2829 | .2300 |
| 5 | .9900 | .9788 | .9666 | .9533 | .8911 | .8178 | .7378 | .6554 | .5738 | .4958 | .4233 | .3574 | .2987 |
| 6 | .9928 | .9847 | .9757 | .9657 | .9176 | .8581 | .7903 | .7174 | .6425 | .5683 | .4967 | .4294 | .3675 |
| 7 | .9948 | .9888 | .9821 | .9746 | .9372 | .8891 | .8322 | .7689 | .7015 | .6324 | .5638 | .4973 | .4343 |
| 8 | .9962 | .9918 | .9867 | .9810 | .9520 | .9132 | .8658 | .8113 | .7516 | .6886 | .6242 | .5601 | .4977 |
| 9 | .9972 | .9939 | .9901 | .9857 | .9631 | .9319 | .8926 | .8462 | .7939 | .7372 | .6779 | .6173 | .5570 |
| 10 | .9979 | .9954 | .9925 | .9892 | .9716 | .9465 | .9141 | .8748 | .8294 | .7791 | .7251 | .6688 | .6115 |
| 11 | .9984 | .9966 | .9944 | .9918 | .9780 | .9579 | .9313 | .8982 | .8591 | .8149 | .7664 | .7147 | .6610 |
| 12 | .9988 | .9974 | .9957 | .9938 | .9830 | .9669 | .9450 | .9173 | .8839 | .8453 | .8021 | .7552 | .7056 |
| 13 | .9991 | .9980 | .9967 | .9952 | .9868 | .9739 | .9560 | .9329 | .9045 | .8710 | .8329 | .7907 | .7454 |
| 14 | .9993 | .9985 | .9975 | .9964 | .9898 | .9794 | .9648 | .9456 | .9215 | .8927 | .8593 | .8217 | .7806 |
| 15 | .9995 | .9989 | .9981 | .9972 | .9920 | .9837 | .9719 | .9559 | .9356 | .9109 | .8818 | .8486 | .8116 |
| 16 | .9996 | .9991 | .9985 | .9978 | .9938 | .9871 | .9775 | .9643 | .9472 | .9261 | .9009 | .8717 | .8387 |
| 17 | .9997 | .9993 | .9989 | .9983 | .9951 | .9898 | .9820 | .9711 | .9568 | .9389 | .9171 | .8916 | .8623 |
| 18 | .9998 | .9995 | .9991 | .9987 | .9962 | .9920 | .9856 | .9766 | .9647 | .9495 | .9308 | .9086 | .8828 |
| 19 | .9998 | .9996 | .9993 | .9990 | .9970 | .9936 | .9885 | .9811 | .9712 | .9583 | .9424 | .9231 | .9005 |
| 20 | .9999 | .9997 | .9995 | .9992 | .9977 | .9950 | .9908 | .9847 | .9765 | .9657 | .9520 | .9354 | .9157 |
| 21 | | .9998 | .9996 | .9994 | .9982 | .9960 | .9926 | .9877 | .9808 | .9717 | .9602 | .9459 | .9287 |
| 22 | | .9998 | .9997 | .9995 | .9986 | .9968 | .9941 | .9900 | .9843 | .9767 | .9669 | .9547 | .9398 |
| 23 | | .9999 | .9998 | .9996 | .9989 | .9975 | .9953 | .9920 | .9373 | .9809 | .9726 | .9622 | .9493 |
| 24 | | | .9998 | .9997 | .9991 | .9980 | .9962 | .9935 | .9896 | .9843 | .9773 | .9684 | .9573 |
| 25 | | | .9999 | .9998 | .9993 | .9984 | .9970 | .99[illegible]3 | .9916 | .9871 | .9813 | .9737 | .9642 |

| x | Mean .25 | .50 | .75 | 1.0 | 2.0 | 3.0 | 4.0 | 5.0 | 6.0 | 7.0 | 8.0 | 9.0 | 10.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | .9143 | .8360 | .7643 | .6988 | .4884 | .3413 | .2385 | .1667 | .1165 | .0814 | .0569 | .0397 | .0278 |
| 1 | .9524 | .9056 | .8599 | .8153 | .6511 | .5119 | .3975 | .3056 | .2329 | .1764 | .1327 | .0994 | .0741 |
| 2 | .9691 | .9376 | .9056 | .8735 | .7461 | .6257 | .5167 | .4213 | .3397 | .2713 | .2149 | .1689 | .1319 |
| 3 | .9786 | .9562 | .9330 | .9091 | .8094 | .7078 | .6095 | .5177 | .4346 | .3610 | .2970 | .2424 | .1962 |
| 4 | .9846 | .9682 | .9509 | .9328 | .8543 | .7695 | .6829 | .5981 | .5176 | .4432 | .3758 | .3158 | .2632 |
| 5 | .9887 | .9764 | .9633 | .9495 | .8871 | .8167 | .7417 | .6651 | .5896 | .5172 | .4493 | .3868 | .3302 |
| 6 | .9915 | .9822 | .9722 | .9615 | .9118 | .8534 | .7890 | .7209 | .6516 | .5830 | .5166 | .4538 | .3953 |
| 7 | .9936 | .9965 | .9787 | .9703 | .9306 | .8820 | .8273 | .7674 | .7047 | .6409 | .5776 | .5160 | .4573 |
| 8 | .9951 | .9896 | .9836 | .9769 | .9451 | .9052 | .8584 | .8062 | .7501 | .6916 | .6322 | .5731 | .5155 |
| 9 | .9962 | .9919 | .9872 | .9820 | .9563 | .9234 | .8838 | .8385 | .7887 | .7357 | .6807 | .6249 | .5693 |
| 10 | .9971 | .9937 | .9900 | .9858 | .9652 | .9379 | .9045 | .8654 | .8216 | .7740 | .7236 | .6715 | .6187 |
| 11 | .9977 | .9951 | .9921 | .9888 | .9721 | .9496 | .9214 | .8878 | .8495 | .8070 | .7612 | .7131 | .6635 |
| 12 | .9982 | .9961 | .9938 | .9912 | .9776 | .9590 | .9353 | .9065 | .8731 | .8354 | .7942 | .7502 | .7040 |
| 13 | .9986 | .9969 | .9951 | .9930 | .9820 | .9666 | .9467 | .9221 | .8931 | .8599 | .8230 | .7829 | .7404 |
| 14 | .9989 | .9976 | .9961 | .9944 | .9855 | .9728 | .9561 | .9351 | .9100 | .8808 | .8479 | .8118 | .7728 |
| 15 | .9991 | .9981 | .9969 | .9955 | .9883 | .9778 | .9637 | .9459 | .9242 | .8987 | .8696 | .8371 | .8017 |
| 16 | .9993 | .9985 | .9975 | .9964 | .9905 | .9819 | .9701 | .9549 | .9362 | .9140 | .8883 | .8593 | .8272 |
| 17 | .9994 | .9988 | .9980 | .9971 | .9923 | .9852 | .9753 | .9624 | .9464 | .9271 | .9044 | .8786 | .8497 |
| 18 | .9996 | .9990 | .9984 | .9977 | .9938 | .9879 | .9796 | .9687 | .9549 | .9382 | .9183 | .8954 | .8696 |
| 19 | .9996 | .9992 | .9987 | .9981 | .9949 | .9900 | .9831 | .9739 | .9621 | .9476 | .9303 | .9101 | .8870 |
| 20 | .9997 | .9994 | .9990 | .9985 | .9959 | .9918 | .9861 | .9782 | .9682 | .9557 | .9405 | .9227 | .9022 |
| 21 | .9998 | .9995 | .9992 | .9988 | .9967 | .9933 | .9885 | .9819 | .9733 | .9625 | .9493 | .9337 | .9155 |
| 22 | .9998 | .9996 | .9993 | .9990 | .9973 | .9945 | .9905 | .9849 | .9776 | .9683 | .9569 | .9431 | .9270 |
| 23 | .9999 | .9997 | .9995 | .9992 | .9978 | .9955 | .9921 | .9874 | .9812 | .9732 | .9633 | .9513 | .9371 |
| 24 | | .9997 | .9996 | .9994 | .9982 | .9963 | .9935 | .9895 | .9842 | .9774 | .9688 | .9583 | .9458 |
| 25 | | .9998 | .9997 | .9995 | .9986 | .9969 | .9946 | .9913 | .9868 | .9809 | .9735 | .9644 | .9534 |

Cumulative probability $\sum_{0}^{\hat{}} P$ for q = 7.0

| x \ Mean | .25 | .50 | .75 | 1.0 | 2.0 | 3.0 | 4.0 | 5.0 | 6.0 | 7.0 | 8.0 | 9.0 | 10.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | .9221 | .8503 | .7841 | .7230 | .5228 | .3780 | .2733 | .1976 | .1429 | .1033 | .0747 | .0540 | .0390 |
| 1 | .9551 | .9110 | .8681 | .8263 | .6721 | .5399 | .4294 | .3387 | .2653 | .2066 | .1600 | .1234 | .0948 |
| 2 | .9698 | .9392 | .9086 | .8800 | .7575 | .6441 | .5410 | .4496 | .3703 | .3025 | .2454 | .1978 | .1585 |
| 3 | .9783 | .9560 | .9332 | .9099 | .8144 | .7185 | .6260 | .5394 | .4602 | .3893 | .3267 | .2722 | .2253 |
| 4 | .9839 | .9671 | .9497 | .9316 | .8550 | .7742 | .6927 | .6131 | .5373 | .4667 | .4021 | .3439 | .2921 |
| 5 | .9878 | .9749 | .9613 | .9471 | .8852 | .8173 | .7461 | .6742 | .6034 | .5354 | .4711 | .4115 | .3570 |
| 6 | .9906 | .9805 | .9698 | .9586 | .9082 | .8511 | .7894 | .7251 | .6601 | .5958 | .5336 | .4743 | .4187 |
| 7 | .9926 | .9847 | .9762 | .9672 | .9260 | .8780 | .8247 | .7677 | .7086 | .6489 | .5896 | .5320 | .4767 |
| 8 | .9942 | .9879 | .9811 | .9738 | .9401 | .8996 | .8537 | .8035 | .7503 | .6953 | .6397 | .5845 | .5306 |
| 9 | .9954 | .9904 | .9849 | .9790 | .9512 | .9171 | .8776 | .8336 | .7859 | .7358 | .6842 | .6320 | .5802 |
| 10 | .9963 | .9923 | .9878 | .9830 | .9601 | .9314 | .8975 | .8589 | .8165 | .7711 | .7236 | .6748 | .6255 |
| 11 | .9970 | .9938 | .9902 | .9862 | .9672 | .9431 | .9139 | .8803 | .8427 | .8019 | .7584 | .7131 | .6667 |
| 12 | .9976 | .9950 | .9920 | .9888 | .9730 | .9526 | .9277 | .8984 | .8652 | .8286 | .7891 | .7473 | .7040 |
| 13 | .9981 | .9959 | .9935 | .9909 | .9778 | .9605 | .9391 | .9137 | .8845 | .8518 | .8160 | .7778 | .7376 |
| 14 | .9984 | .9967 | .9947 | .9925 | .9816 | .9671 | .9487 | .9267 | .9010 | .8719 | .8397 | .8048 | .7677 |
| 15 | .9987 | .9973 | .9957 | .9939 | .9848 | .9725 | .9568 | .9376 | .9151 | .8893 | .8604 | .8288 | .7947 |
| 16 | .9990 | .9978 | .9964 | .9949 | .9874 | .9770 | .9635 | .9470 | .9272 | .9044 | .8785 | .8500 | .8189 |
| 17 | .9991 | .9982 | .9971 | .9958 | .9895 | .9807 | .9692 | .9549 | .9376 | .9174 | .8944 | .8686 | .8403 |
| 18 | .9993 | .9985 | .9976 | .9966 | .9913 | .9838 | .9740 | .9616 | .9465 | .9287 | .9082 | .8851 | .8594 |
| 19 | .9994 | .9988 | .9980 | .9971 | .9927 | .9864 | .9780 | .9673 | .9542 | .9385 | .9203 | .8996 | .8764 |
| 20 | .9995 | .9990 | .9983 | .9976 | .9939 | .9886 | .9814 | .9722 | .9607 | .9470 | .9308 | .9123 | .8914 |
| 21 | .9996 | .9991 | .9986 | .9980 | .9949 | .9904 | .9843 | .9763 | .9663 | .9543 | .9400 | .9234 | .9046 |
| 22 | .9997 | .9993 | .9989 | .9984 | .9957 | .9919 | .9867 | .9798 | .9711 | .9606 | .9479 | .9332 | .9164 |
| 23 | .9997 | .9994 | .9991 | .9986 | .9964 | .9932 | .9887 | .9828 | .9753 | .9660 | .9549 | .9418 | .9257 |
| 24 | .9998 | .9995 | .9992 | .9989 | .9970 | .9943 | .9904 | .9853 | .9788 | .9707 | .9609 | .9493 | .9358 |
| 25 | .9998 | .9996 | .9993 | .9991 | .9975 | .9952 | .9919 | .9875 | .9818 | .9747 | .9661 | .9558 | .9438 |

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